

Principles of Animation Physics

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Introduction

Why should a character animator care about physics?

If you're going to be a surgeon then you need to know advanced anatomy; for life drawing a basic knowledge of muscles and bones is helpful. If you're going to be an engineer then you need to know calculus and physics; as a character animator a basic understanding of mechanics and bio-mechanics is helpful. In *Chuck Amuck*, Chuck Jones writes, "Comparative anatomy is a vital tool of the complete animator or director." The purpose of this course is to make physics another tool in your animator's toolbox.

At San Jose State I teach *Physics of Animation*, a one semester course for animation artists. The first eight weeks of the semester cover mechanics and bio-mechanics; this SIGGRAPH course is a distillation of those 16 lectures down to a three hours. The focus of today's course is character animation, though many of the concepts are also useful for character effects, such as hair and clothing. Physics is also useful for effects animators and for lighting artists but we won't have time to get to those areas today.

All animators are familiar with the "Principles of Animation" as described by Ollie Johnston and Frank Thomas in their book *The Illusion of Life: Disney Animation*. These principles, such as "Squash and stretch" and "Anticipation", give helpful guidance to animators. And since they describe motion it's not surprising that many of the principles of animation are based on physical laws. For example, the animation principle of follow-through is based on the Law of Inertia.

We'll follow the successful model introduced by Johnston and Thomas and make a list of principles to organize our understanding of physical motion. Specifically, our Principles of Animation Physics will be:

1. Timing, Spacing, and Scale
2. Law of Inertia
3. Momentum and Force
4. Center of Gravity
5. Weight Gain and Loss
6. Action-Reaction

In the first half of the course I'll explain each principle and how it applies in simple situations. Then in the second half we'll use these principles to break down and understand such actions as jumping and walking.

One last thing before we start: These principles of animation physics are *not* for the purpose of creating physically accurate motion. They are guides to help you interpret and understand physical motion. Animators commonly use video reference, not to copy it but to extract the essence that they need. Using physics you'll be able to imagine scenarios, such as a giraffe on a tightrope or a lion on trapeze, that you'll never find on YouTube. To quote Chuck Jones once more, "We must all start with the believable. That is the essence of our craft."

Principles of Animation Physics

Timing, Spacing, and Scale

The first exercise that animators practice on is usually the bouncing ball, learning how to adjust the timing and spacing so that it slows into and out of the apex in a believable way. Timing and spacing indicate speed and when spacings change there's acceleration. Because the acceleration of gravity is constant the ball's spacings follow a simple pattern. I call this pattern the "Odd Rule" because the spacings from the apex go as 1, 3, 5, 7, 9, and so on. Figure 1 shows other examples of slowing out and slowing in using the Odd Rule. This pattern isn't a simplified rule-of-thumb; the Odd Rule is as mathematically precise as $x = \frac{1}{2}at^2$.

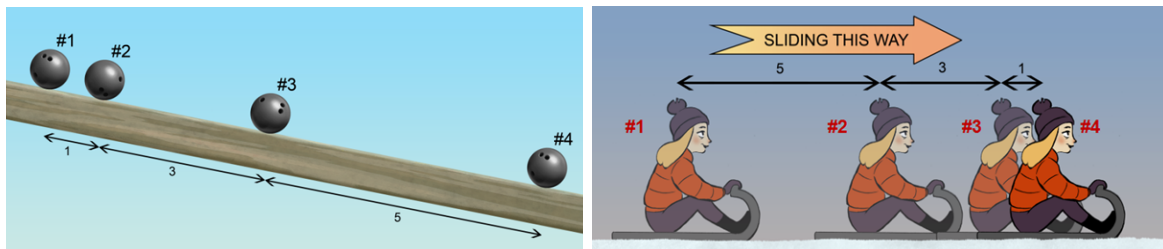


Figure 1: Examples of the Odd Rule for slowing out (left) and slowing in (right).

The Odd Rule is helpful for animating straight ahead action but for pose-to-pose animation there's a useful corollary that I call "Fourth Down at Half Time."* The rule is very simple: the position of the breakdown pose, between the apex and the final pose, is a quarter of the distance from the apex; Figure 2 shows some examples. As with the Odd Rule, Fourth Down at Half Time applies whenever acceleration is constant, that is, when forces are constant.

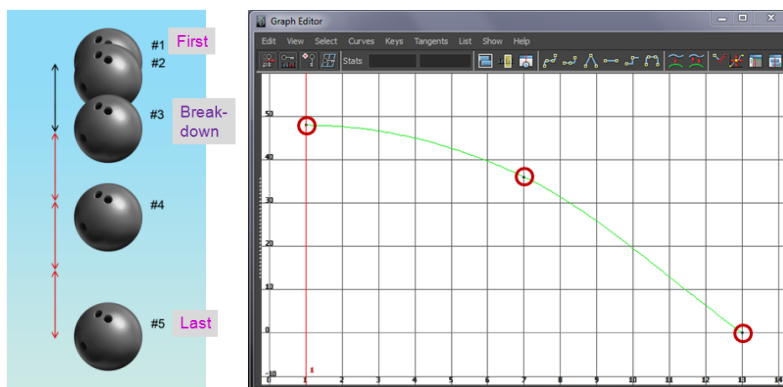


Figure 2: (Left) An example of Fourth Down at Half Time; the timing is three frames per drawing. (Right) Using Fourth Down at Half Time in the graph editor.

For acceleration under gravity the distance fallen from the apex equals $(\frac{1}{3}$ inch) times the square of the number of frames.[†] For example, for 6 frames the distance fallen is $\frac{1}{3} \times (6)^2 = 12$ inches or one foot; for 12 frames the distance fallen is $\frac{1}{3} \times (12)^2 = 48$ inches or four feet.

These numerical calculations are rarely useful to character animators but I'm mentioning them because of the importance of timing in creating scale. Suppose that a character jumps off the roof of a house and lands on the ground in 12 frames. The character will feel small because this timing indicates that the house is four feet tall. This applies not just to falling motion but also walking, running, jumping, etc. since timing of the arms and legs has a similar scaling. This connection between timing and scale is used in live-action movies when scale models are filmed with high-speed cameras; by filming at twice the normal speed the models look four times bigger when the action is played back at normal speed. It's important to remember that in order to "stay on model" the timing of a character's action must be consistent with the character's size. Just a 20% error in the timing will make a 6 foot character seem like it's only about 4 feet tall.

Law of Inertia

The Law of Inertia, also known as Newton's First Law of Motion, says that a character will move with constant velocity unless acted on by an unbalanced force. First, let's be clear what's

*This mnemonic makes more sense if you know American football.

[†]This is for 24 frames per second, as used in film; note that video typically uses 30 fps.

meant by an “unbalanced force.” The simplest form of the Law of Inertia is that an object in motion remains in motion unless acted on by a force, for example, an asteroid in deep space moves with constant velocity. But unless you animated a shot in *Wall-E* it’s unlikely you’ve encounter a situation where there’s absolutely no forces.

A rock sitting on a table has two forces on it: gravity pulling downward and the table surface pushing upward. These two forces balance each other. A bowling ball rolling on a smooth floor has a similar pair of balanced forces (gravity and the floor). If there’s no other force on the bowling ball then there’s no unbalanced forces so it moves with constant velocity. Same with the rock, it’s speed just happens to be zero.

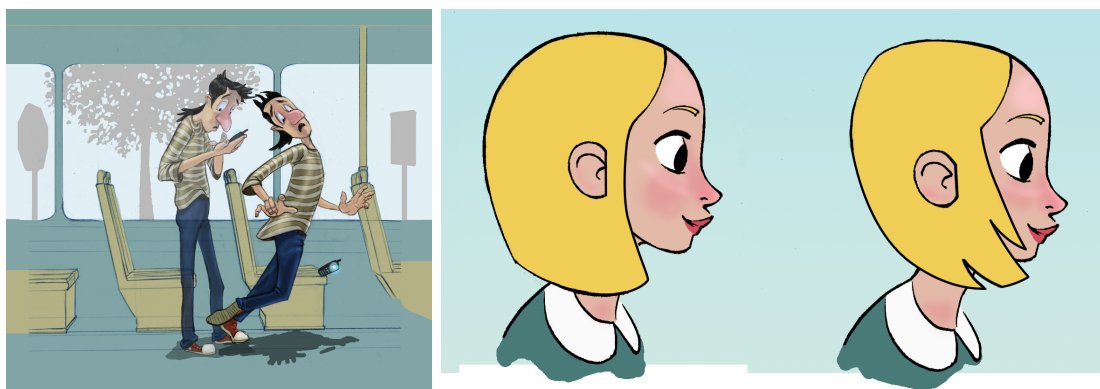


Figure 3: (Left) Character demonstrating the Law of Inertia by standing on a bus when it comes to a sudden stop. (Right) This character is seated when the bus stops; notice how her hair flies forward due to the Law of Inertia.

At first this all seems purely academic until you realize that follow-through in animated motion is entirely due to the Law of Inertia. Let’s take a simple example of a character standing on a bus (see Figure 3). When the bus suddenly stops the character goes flying forward. Before the bus hit the brakes he was moving and by the Law of Inertia, he’ll continue moving until a force acts to stop him (such as when he hits the floor). Another example is seen in the follow-through (forward motion) of a seated passenger’s hair (see Figure 3 again) when the bus hits the brakes.

A corollary of the Law of Inertia is that a character at rest will remain at rest until acted on by an unbalanced force. If the bus suddenly accelerates forward then our standing character falls on his back (see Figure 4) and the seated passenger’s hair seems to drag behind. As fellow passengers it seems to us as if there’s a force pulling everything backwards but that’s because we’re moving with the bus. A stationary observer standing outside the bus would realize that the poor chap that’s falling has the bus moving out from under him (see Figure 5). This reminds us that you have to be careful to include the effect of the camera’s motion, especially when the camera is accelerating.

Finally, the Law of Inertia also explains the drag and outward “centrifugal” force that’s

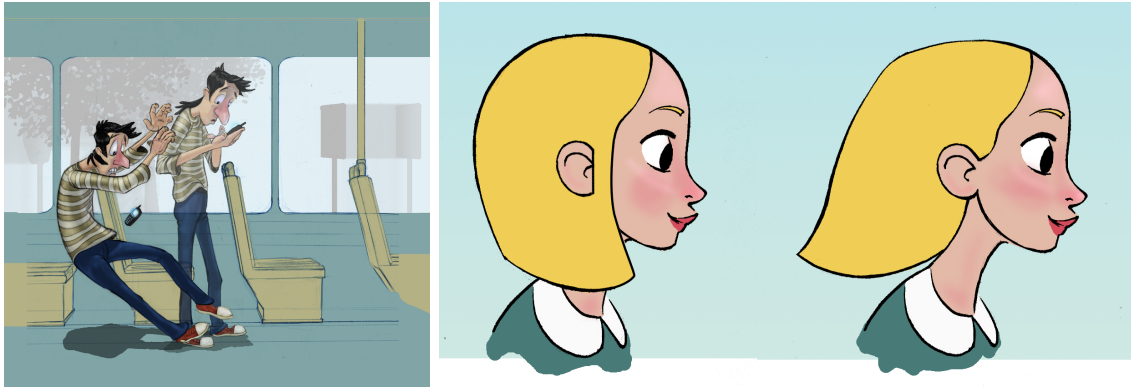


Figure 4: (Left) The standing character falls when the bus accelerates forward. (Right) This character is seated when the bus accelerates; notice how her hair drags behind due to the Law of Inertia.



Figure 5: When the bus suddenly accelerates the character appears to fall backwards, as seen from inside the bus (left), but an observer outside the bus sees the bus moving out from under his feet (right).

illustrated in Figure 6. As the characters turn their bodies their hair and clothing drag behind due to the Law of Inertia (an object at rest will remain at rest until acted on by an unbalanced force). Once the hair and clothing start moving they'll continue moving, causing them to fly outward due to inertia (or, if you prefer, due to follow-through). In the same fashion, if the bus makes a sudden right turn then the character standing on the bus will continue moving in his original direction, causing him to fall towards the left side of the bus.

Momentum and Force

A moving character has momentum, which depends on the character's speed and on its weight. A character that weighs 100 pounds can have as much momentum as a 300 pound character if

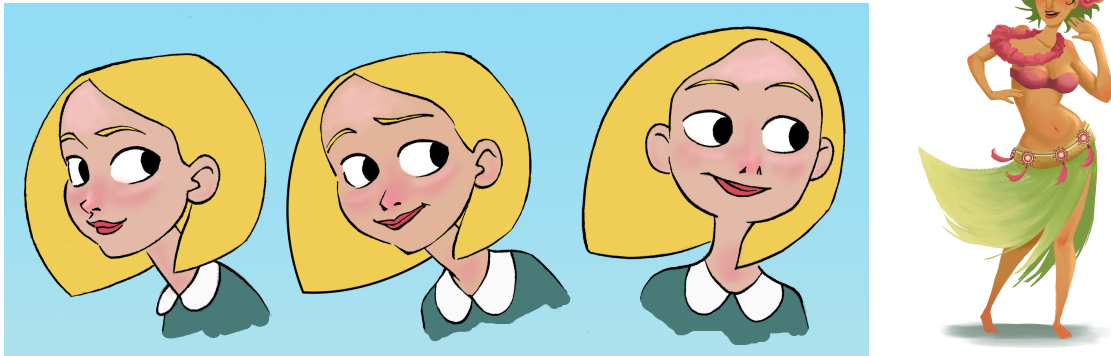


Figure 6: (Left) As she turns her head, the character’s hair drags behind but also flies outward. (Right) Hula skirt and lei necklace drag behind but also fly outward as the character turns her body.

the small character runs three times faster than the big guy. Momentum also depends on the direction of the motion so deflecting a moving object is considered a change of momentum.

To change a character’s momentum you need an unbalanced force; the bigger the force, the quicker the momentum changes. This change in momentum could be slowing or increasing a character’s speed or it could be a change in direction (such as making a turn). In mechanics this is known as Newton’s Second Law of Motion.

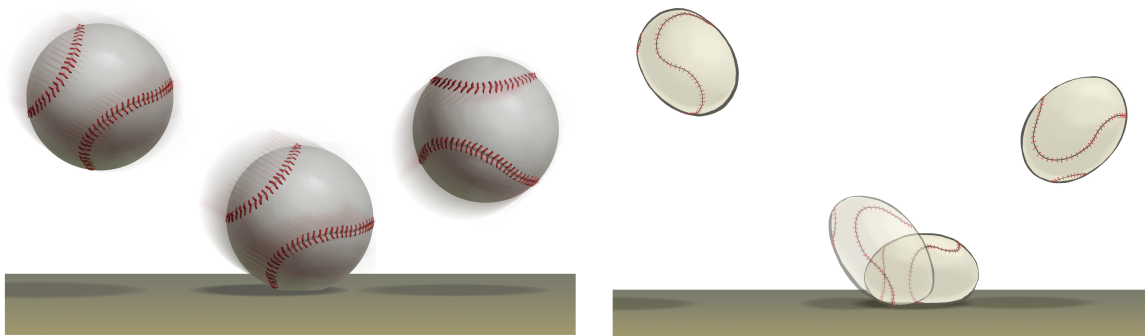


Figure 7: (Left) Momentum changes quickly on impact so the force of impact is large. (Right) Momentum changes more slowly so the force is softer, the impact is less jarring.

Figure 7 shows two examples of a ball hitting the ground. The momentum change could be about the same in the two cases but in the realistic impact that change occurs quickly, implying a large force at impact. Using “squash and stretch” the cartoony version softens the impact by

having the change in momentum spread over a few frames. In brief, crisp timing means large forces and vice versa.

Figure 1 shows a character sliding to a stop due to friction (the unbalanced force). If the friction is small then the momentum changes slowly; if the character starts with a large momentum then the character slides for a long time before coming to a stop. And if the force is constant (which is typical for sliding friction) then the change in momentum produces spacings that follow the Odd Rule and Fourth Down at Half Time, coming to a stop at the “apex.”

Figure 8 shows examples where the unbalanced force is not constant. If the force is increasing, such as when a ball rolls down a steepening hill or when an object tips over, then the spacings increase faster than the Odd Rule. On the other hand, if the unbalanced force is decreasing then the spacings become more uniform. For example, at the bottom of its swing the speed of a pendulum is nearly constant since the tension in the rope almost balances the force of gravity.

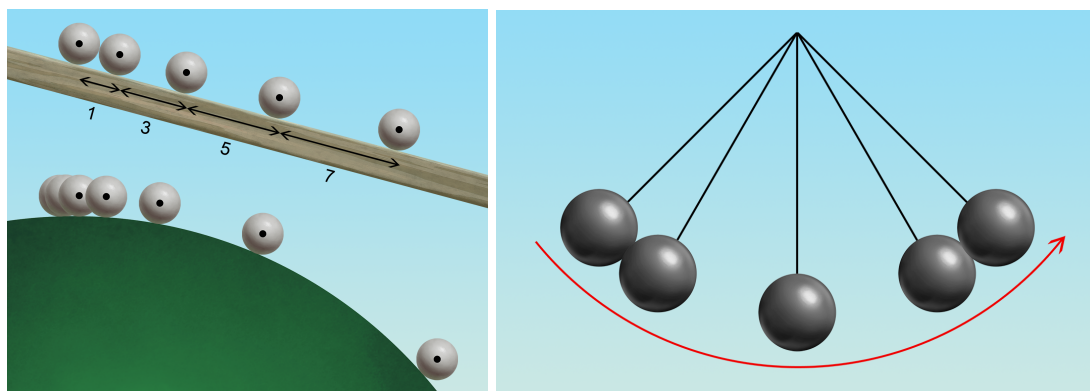


Figure 8: Examples where the unbalanced forces are not constant. Although the force of gravity is constant, the forces exerted by the surface of the curved ramp (left) and the tension in the pendulum (right) depend on the position of the ball.

Finally, if the unbalanced force goes to zero then the velocity becomes constant (see the Law of Inertia). An example of this is when a falling character reaches terminal velocity because the force of gravity is balanced by the force of air resistance.[‡] In brief, the texture of the timing depends on the unbalanced forces and how those forces change during the action. The animated motion is believable only if the texture is consistent with the forces.

Center of Gravity

The center of gravity is the average location of an object’s weight. This is the geometric (visual) center for most objects but for non-uniform objects (e.g., hammer with a wooden handle and an

[‡]For humans this maximum speed is over 100 miles per hour but for small creatures, such as squirrels, it’s quite slow so they survive falls from any height.

iron head) it's located closer to the heavier side. A person's center of gravity is roughly located in the center of the torso, at about the height of the belly button, but it can shift depending on the pose (see Figure 9).

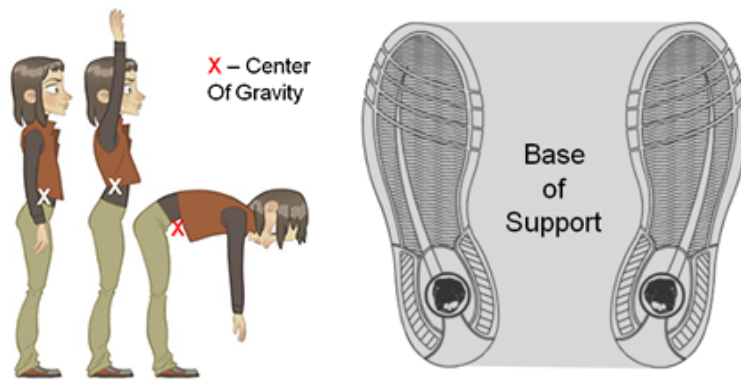


Figure 9: (Left) Location of a character's center of gravity in various poses. (Right) Shaded area is the character's base of support.

The center of gravity is important for various reasons. First, a character is in a balanced stationary pose only if the center of gravity is positioned over the character's feet. More precisely, the center of gravity has to be over the "base of support", which is the area around the feet (see Figure 9 again).

Second, when a character changes pose the center of gravity shifts, causing a weight shift from one leg to another. Simply shifting the center of gravity by a few inches is enough to cause significantly weight shift, as shown in Figure 10. Weight shifts from foot to foot are reflected in the pose, typically raising the hip and lowering the shoulder on the weight-bearing side (an effect known as "contrapposto"). When animated well it should be clear whether a character is standing or sitting even if the shot only shows the character's upper body.

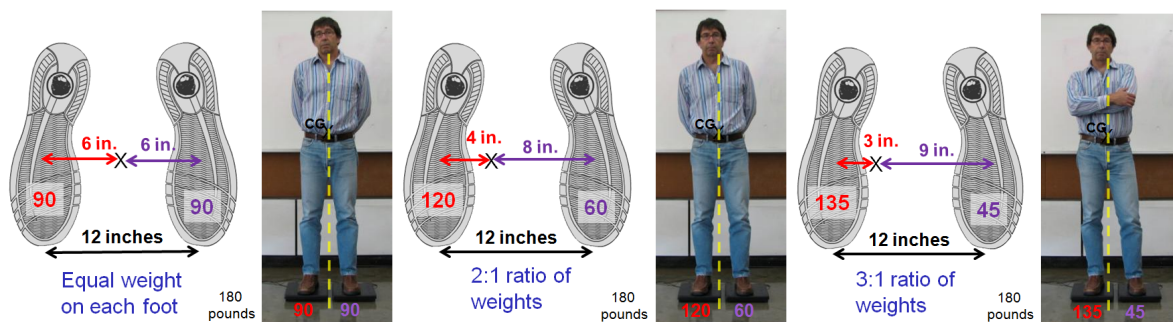


Figure 10: (Left) Even weight on each foot when the center of gravity is centered above them. (Center and Right) Weight shift due to shift of center of gravity towards screen left.

Finally, when we consider the path of action of a moving character the point that we're most interested in tracking is the center of gravity. For example, when flying through the air it is the center of gravity that follows a parabolic arc, independent of any rotation in the body or twisting of the limbs (see Figure 11). The motion of the center of gravity is also important in walks, as we'll discuss later.

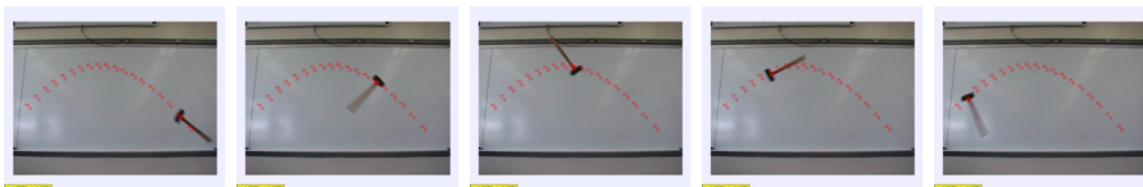


Figure 11: Path of action of the center of gravity of a spinning hammer flying through the air.

Weight Gain and Loss

We think of a character's weight as being constant but when the character accelerates its weight effectively varies. For example, as illustrated in Figure 12, if the character is:

- Moving upward and gaining speed: Gain weight
- Moving upward and losing speed: Lose weight
- Moving downward and gaining speed: Lose weight
- Moving downward and losing speed: Gain weight

In brief, if you're going against gravity (rising but speeding up or falling but slowing down) then you gain weight but if you're going with gravity (rising & slowing down or falling & speeding up) then you lose weight. An extreme case is when a character is in free-fall and becomes weightless. You can feel these variations of weight by taking a heavy object in your hand, say a dumbbell, and accelerating it up and down.[§]

As a character moves (walks, runs, jumps, etc.) the changes in weight create many overlapping actions in the movement of hair, clothing, and flesh. Gaining weight pulls these downward while losing weight causes them to almost float. Richard Williams calls this effect "counteraction" and in *The Animator's Survival Kit* he writes, "When the character (accelerates) up the drapery or hair or soft bits go down." Poorly animated characters sometimes look "floaty" as they walk because their actions lack this variation in weight.

[§]If your physics friends question this weight gain and loss just tell them that you're working in a non-inertial frame of reference.

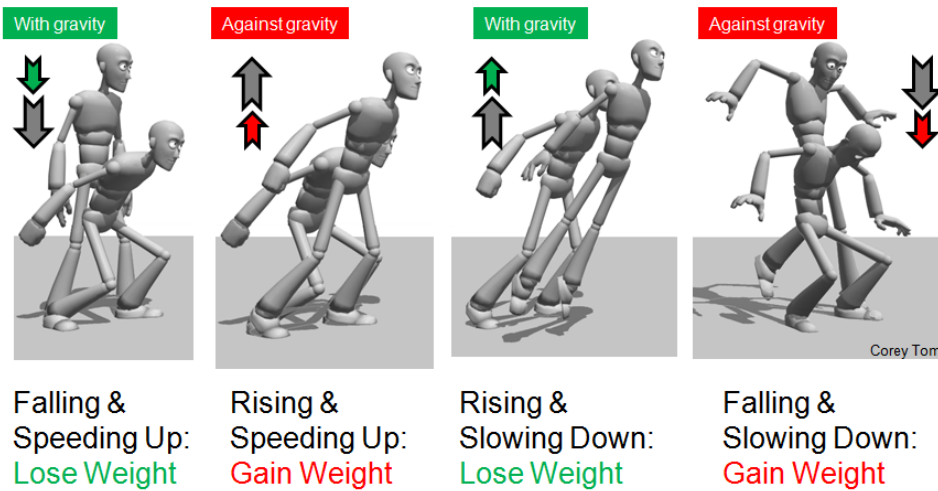


Figure 12: Weight changes as a character: crouches down, pushes upward, flies in the air, and settles on landing.

Action-Reaction

The principle of action-reaction, also known as Newton’s Third Law, tells us that when a character interacts with an object (or with another character) both are affected. Paul Hewitt poetically describes this as, “You cannot touch without being touched.” A more traditional statement of the principle would be,

For every “action” force there’s an instantaneous “reaction” force that’s equal in magnitude and opposite in direction.

Let’s consider each part of this statement. First, the action and the reaction are a pair of matched forces. For example, suppose Mr. Alpha punches Mr. Beta (see Figure 13). The action is the force of Mr. Alpha’s fist hitting Mr. Beta’s face so the reaction is the force of Mr. Beta’s face pushing back on Mr. Alpha’s fist.[¶] Second, these two forces are instantaneous so if Mr. Beta punches back that’s not the reaction, that’s a new action.

Furthermore, if Mr. Alpha punches with 100 pounds of force towards the left then there’s an equal force of 100 pounds on his fist towards the right. Since forces change momentum the action force causes the Mr. Beta’s head to start moving towards the left while the reaction force changes the momentum of Mr. Alpha’s fist, possibly bringing it to a stop. To animate this scene successfully it’s essential to match the action and reaction. A common mistake is to focus on animating Mr. Beta’s motion while neglecting the reaction that must simultaneously be occurring on Mr. Alpha.

[¶]It doesn’t matter which force is labeled “action” and which one is “reaction”; you can always switch the names since they’re symmetric.

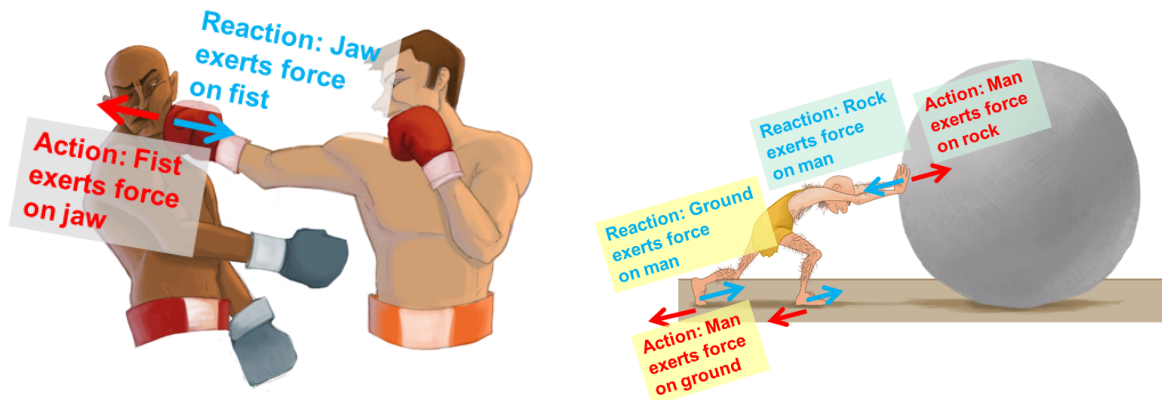


Figure 13: Examples of action-reaction forces: (Left) Mr. Alpha punching Mr. Beta. (Right) Man pushing a rock.

Judging the effect of action-reaction is complicated by the fact that there's usually several action forces to be considered. Take the simple case of a man pushing a rock (see Figure 13). The man exerts a force on the rock so the rock exerts a force back on him. If he was on roller skates he'd move backwards and the rock would move forward. But he's barefoot so there's another set of action-reaction forces: he also pushes back on the ground and the ground pushes him forward. In order for him to push the rock forward the force exerted by his legs cannot be less than the force exerted by his arms. These actions and reactions have to appear to match in order to animate this scene believably.

Finally, the action-reaction principle is often violated in films, both animated and live-action. Heros deliver powerful punches (or shoot big guns) with negligible recoil while the villain goes flying backwards. This may be done intentionally for dramatic effect (to make the hero look powerful) or for comic effect (to defuse the violence) but if it's unintentional and unexpected then there's a chance of shattering the audience's suspension of disbelief.

Applying the Principles

Basic Poses

In this second half we'll apply the principles of animation physics to a variety of examples, starting with basic poses. The principle of the Center of Gravity tells us that in a stationary pose the character's center of gravity is located over its base of support (area around the feet; see Figure 9). To check this it's helpful to draw the "line of gravity", the vertical line passing through the center of gravity, and the "center of pressure", which is where the line of gravity hits the floor (see Figure 14). If the center of pressure falls inside the base of support then the force of gravity is balanced by the upward force of the floor.

Let's see how this applies in some examples: A character has to lean when carrying a heavy

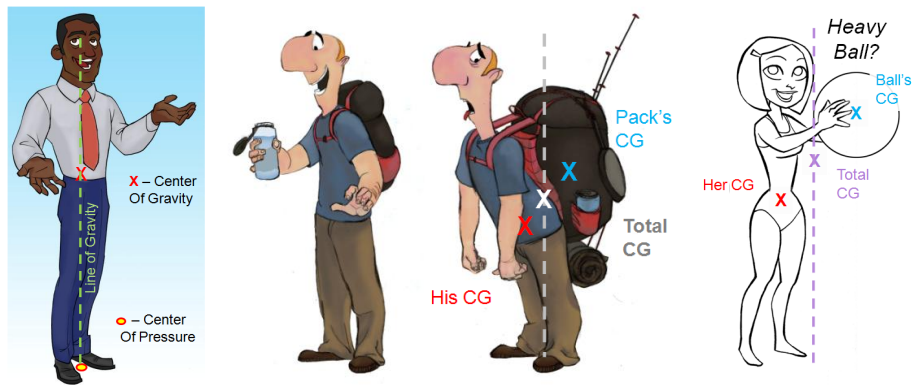


Figure 14: (Left) Line of gravity and center of pressure. (Center) Light and heavy backpacks. (Right) Character would be out of balance if the ball were heavy.

weight, such as the backpacker in Figure 14, to keep the combined center of gravity above the base of support. The woman in Figure 14 cannot be carrying a heavy ball because, regardless of her strength, if the ball were actually very heavy then the combined center of gravity would be in front of her, outside her base of support.

Notice the difference in the poses for characters lifting light and heavy objects shown in Figure 15. The character lifting the heavy object has to keep it close to her body to position the combined center of gravity over her feet. If she lifts it quickly (accelerating it upward) then it's even harder for her to stay in balance since the object effectively has greater weight (see the principle of Weight Gain and Loss), which shifts the combined center of gravity.



Figure 15: Characters lifting a light beach ball (left) and a heavy water bottle (right).

The character picking up her sunglasses in Figure 16 has to move her hips backwards as she leans forward in order to maintain her center of gravity over her feet. In this same figure the character rising from a chair leans forward to position his center of gravity over his feet; he also shifts his feet to broaden his base of support. Figure 16 also shows me reaching as far as

possible over an edge. To keep my center of gravity over my foot the other leg is extended as far back as possible. In general, when the base of support is narrow we instinctively raise our arms to our sides since moving them allows us to rapidly reposition our center of gravity.

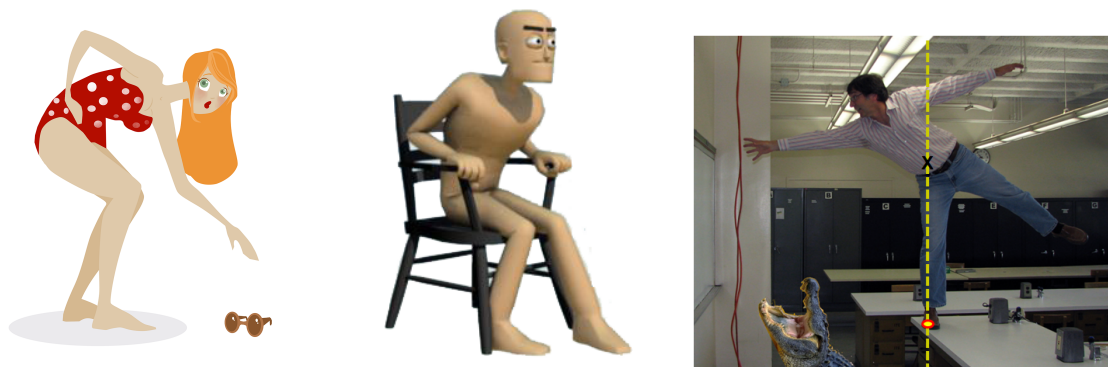


Figure 16: (Left) Character shifts her hips backwards as she leans forward. (Center) Character leans forward and shifts his feet as he rises from the chair. (Right) Reaching out over an edge.

In exceptional cases the line of gravity is not vertical. In Figure 17, as the train accelerates the line of gravity tilts forward so the woman leans forward to stay in balance. In the same figure the distracted bus passenger is out of balance when the bus accelerates; the tilting of the line of gravity moves the center of pressure out from under his base of support. Finally, when a moving character turns a corner there's a change of momentum due to a centripetal force (see the principle of Momentum and Force). The resulting acceleration tilts the line of gravity towards the inside of the curve. For this reason a runner leans into a turn, also shown in Figure 17.

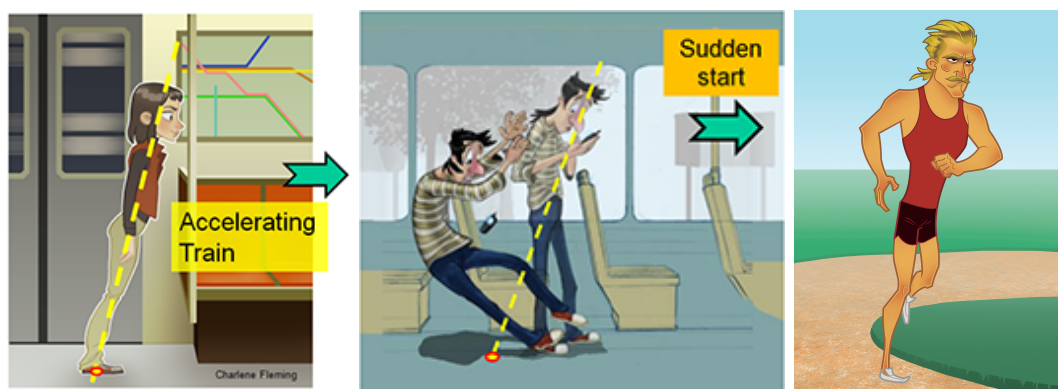


Figure 17: (Left) Maintaining balance on an accelerating train. (Center) Losing balance on an accelerating bus. (Right) Runner leaning into a turn.

Jumps

We'll start with the easiest part of a jump, which is when the character is in the air. For human-sized characters the force of air resistance is small when traveling below 30 miles per hour so typically the only significant force during a jump is gravity. In that case the path of action is a parabolic arc, as shown in Figure 18. The horizontal spacings are uniform and the vertical spacings are given by the Odd Rule and Fourth Down at Half Time (see the principle of Timing, Spacing, and Scale). In the rare cases where air resistance is significant the path of action is skewed, with the descent being at a steeper angle than the ascent.

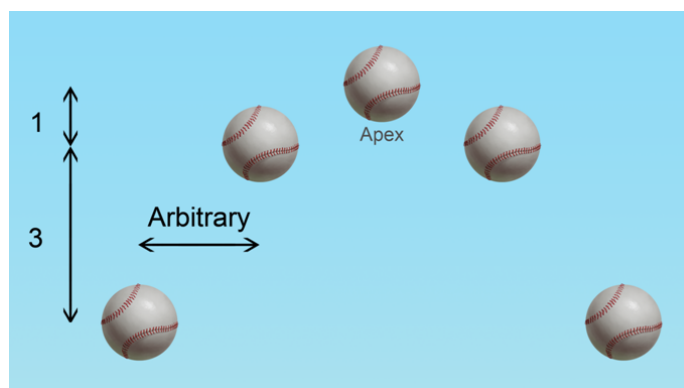


Figure 18: Parabolic arc has uniform horizontal spacings and vertical spacings that follow the Odd Rule and Fourth Down at Half Time.

The timing of the character while in the air is also simple since gravitational acceleration is the same for all objects. For example, everything falls one foot in six frames so jumping one foot in height takes six frames from take-off to apex and another six frames from apex to landing. In Figure 19 this corresponds to a “jump height” of one foot and a “jump time” of six frames. Timing and scale are connected so a four foot jump takes twice as long. Notice that the timing only depends on the (vertical) height of the jump, not on the (horizontal) distance jumped.

Now let's consider the start of a jump, when the character is rising out of the crouch, pushing with its legs in order to get into the air. We'll call the distance over which the character pushes off the “push height” and the distance from take-off to apex the “jump height” (see Figure 19). The ratio of these heights is the “jump magnification”,

$$\text{Jump Magnification} = \frac{\text{Jump height, from take-off to apex}}{\text{Push height, from crouch to take-off}}$$

For example if the character rises from a one foot crouch and jumps four feet into the air then the jump magnification is 4.

The reason that the jump magnification is important is that it determines the timing of the push. The larger the jump magnification, the quicker the push needs to be in order for the

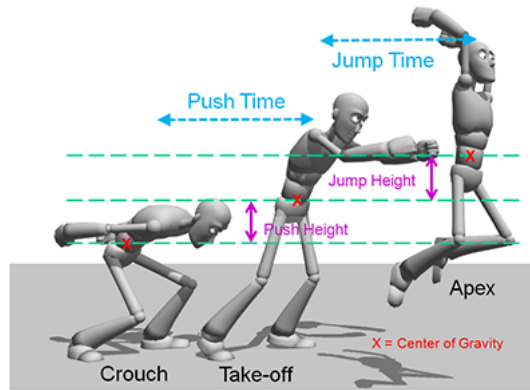


Figure 19: Elements of a jump from start to apex; the second half, from apex to settle, is similar. The character's position is measured from its center of gravity.

jump to look believable. Specifically, if the character pushes with constant force then,

$$\text{Push time, from crouch to take-off} = \frac{\text{Jump time, from take-off to apex}}{\text{Jump Magnification}}$$

The jump time, from take-off to apex, for a four foot jump is 12 frames so if the character crouched one foot then the timing for the upward push should be 3 frames (since jump magnification is 4). This calculation is not as important as the concept that it illustrates: The timing when a character is pushing off the ground needs to be quick if the jump is high. When a powerful super-hero jumps exceptionally high the physically accurate push time could be less than one frame. This is why such scenes are sometimes presented as occurring in slow-motion.

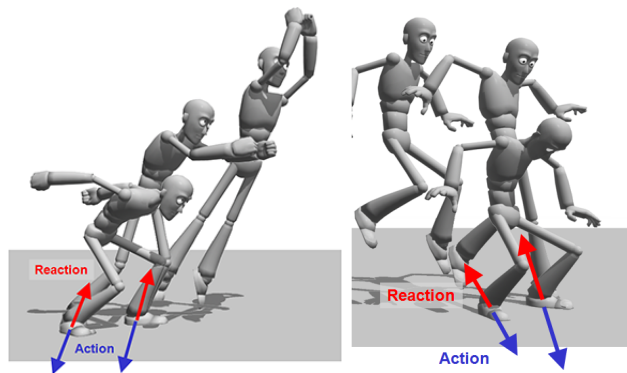


Figure 20: Forces during a jump when taking off (left) and landing (right).

With great force comes great momentum change (and vice versa). The force that a character exerts during the push equals the jump magnification times the character's weight. The character exerts this action force down on the floor and so it's the reaction force that actually

accelerates the character upward (see Figure 20 and the principle of Action-Reaction). This force increases significantly (and the jump is higher) if the character swings its arms upward while pushing off on the ground. However, if the arm swing occurs after the character leaves the ground then the reaction force pushes the torso downward, reducing the height of the jump. Be sure to try this yourself; once you experience the effect of doing it wrong you'll never forget to match the arm swing with the leg push when animating a jump.

Finally, the timing of the settle, when the character lands, is related to the timing of the push. The two have similar timing if the distance that the character crouches on landing is similar to the distance it crouched when pushing off. If the crouch is deeper on the landing then the timing of the landing is slower. On the other hand, if the character lands with straight, stiff legs then the timing is quicker and the impact force is jarring (see the principle of Momentum and Force).

Walks

The bio-mechanics of walks, even just bipeds, could easily be a full three hour course by itself. Rather than trying to cover everything about character walks, we'll focus on examples that illustrate our six principles.

Walking is a cycle of steps alternating between the contact pose and the passing position (see Figure 21). Although there is great variation in walks, the center of gravity generally rises as the character transitions into the passing position and it falls as the character leaves it. This causes the forward motion to slow a bit approaching the passing position and speed up again on leaving the passing position (see Figure 21 again). The texture in this timing is subtle but the variation in speed is easily demonstrated by carrying a shallow pan of water while walking.

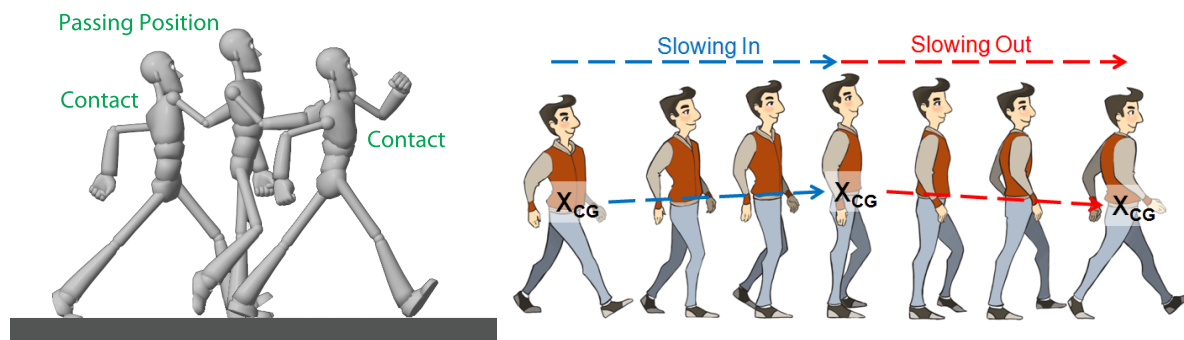


Figure 21: (Left) Contact pose and passing position in a walk. (Right) Slowing into and slowing out of the passing position with the rising and falling of the center of gravity.

As with jumping, the character exerts action forces on the ground and relies on the resulting reaction forces to walk. From the contact pose to the passing position the force on the character is in the backward direction, slowing the motion. The necessity of this backward force is demonstrated by having the character step on a banana peel. From the passing position to the

contact pose the force is in the forward direction, as expected. Interestingly, over the course of a full cycle the forces in the backward direction equal the forces in the forward direction. We know this because the speed is the same at the start and end of the cycle so there's no change in momentum so the total force is zero.

Raising the center of gravity while walking requires effort. This is best appreciated by walking for five minutes as they do in Monty Python's "The Ministry of Silly Walks" sketch; silly walking is surprisingly aerobic exercise. There are various subtle movements that the body performs to minimize the work required while keeping a constant step length and cadence. First, the pelvis rotates to bring the forward leg out during the contact pose (see Figure 22), which keeps the center of gravity from dipping as far down as without the rotation. To allow this pelvic rotation to occur with minimal effort by the legs, the shoulders simultaneously rotate in the opposite direction.^{||}

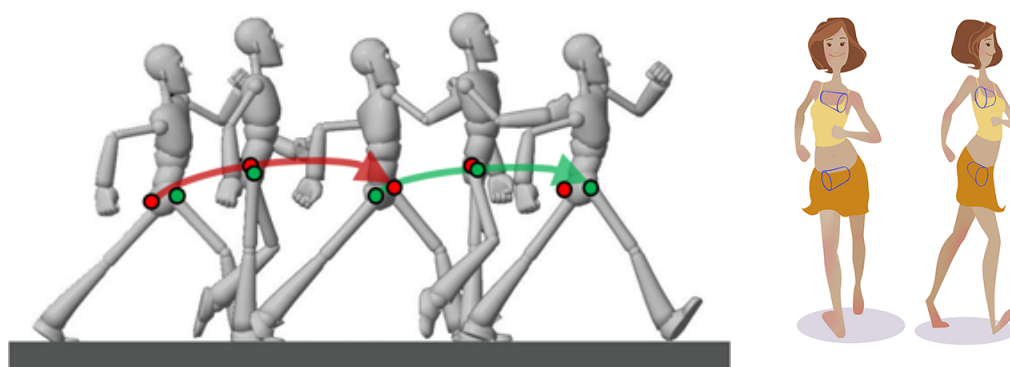


Figure 22: (Left) Pelvic rotation in the contact pose. (Right) Synchronization of hip and shoulder rotations.

To keep the center of gravity from rising very much during the passing position the pelvis tilts to the side of the moving leg; this is called the "pelvic list" (see Figure 23). The knee on the weight-bearing leg bends slightly for the same reason while the knee of the passing leg bends to keep the foot from dragging the ground.

Besides the up-and-down motion of the center of gravity there is also some side-to-side motion. Walking is most efficient when this motion is minimized since forces have to be exerted by the legs to maintain it. Nevertheless there is always some side-to-side motion and it is synchronized with the up-and-down motion. For slow walks the center of gravity traces out an ∞ -shaped pattern, as seen when the character is walking away from the camera. For fast walks the synchronization changes and the center of gravity swings almost like a pendulum, tracing a U-shaped pattern.

Figure 24 shows the weight shift from one foot to the other while walking. The two curves

^{||}To fully explain this rotation we need more physics principles, as discussed in the Concluding Remarks section.



Figure 23: (Left) Pelvic list in the passing position. (Right) Side-to-side motion of the center of gravity.

intersect where weight is transferred from the back foot to the front foot during the contact pose. Notice that the weight varies significantly during the cycle, ranging from about 220 pounds down to 140 pounds. The dips in the weight occur during the passing position as the character slows while rising then speeds while falling (see the principle of Weight Gain and Loss). The spikes in the weight occur on the back foot just before the heel strike of the front foot and on the front foot just as the back foot leaves the ground.

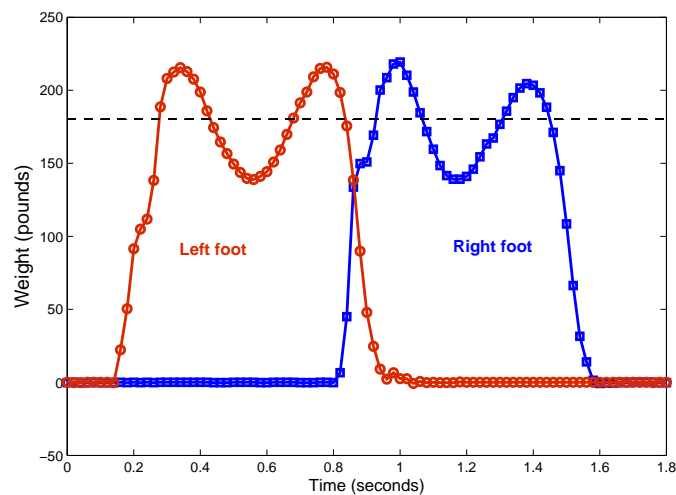


Figure 24: Laboratory measurements of the weight on each foot while walking. The dashed line is the stationary weight (180 pounds).

The data in Figure 24 is for an average walk (about one second per cycle). For slower walks the weight variations are smaller; for very slow walks the weight on the planted foot is

nearly constant from heel strike to toe lift. For brisk walks the weight can vary dramatically, dipping to almost zero during the passing position and spiking to over twice the normal weight around the contact pose. These weight variations are noticeable in the overlapping action of the character's hair, clothes, flesh, etc.

Closing Remarks

Two closing thoughts on physics as it applies to animation: First, while this course covers what I believe to be the most important principles of animation physics, if we had more time then we would add three more to our list. Specifically,

7. Work and Energy
8. Rotational Inertia
9. Angular Momentum and Torque

We already touched on these briefly in the discussion on walks. For example, it takes work to raise the center of gravity and torque to rotate the hips and shoulders. And understanding rotational inertia is essential for more complicated character rotations, such as a back somersault or a cat twisting to land on its feet (see Figure 25). So there's an opportunity for a sequel.

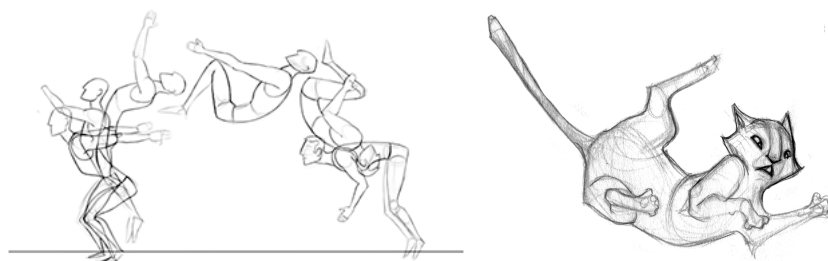


Figure 25: Rotation in a back somersault (left) and a cat twist (right).

Second, the principles of animation physics are useful at all stages, including story, layout, and character design. In the early stages of production the director works with a variety of artists on the visual design (vis-dev). Besides the general look of the film there are many decisions regarding the universe in which the story takes place. For example, animals talk in *Kung Fu Panda* but they don't in *How to Train Your Dragon*.

Similarly, there is also the physical design (phys-dev) of the film's universe. Take, for example, the character of Mantis in *Kung Fu Panda*. In order for Mantis to fight villains who are a hundred times his size the principle of action-reaction is suspended. This decision leads to other phys-dev questions: Does action-reaction apply to the other heros? What about the villains or ordinary citizens? Are there scenes where Mantis will obey action-reaction?

One can also think of a “physics script”, similar to a color script, that indicates the variation in the physics in each scene. For comedic action the laws of physics are often bent but for dramatic action a heightened realism creates more tension. Many of these phys-dev decisions are made instinctively by the director and when the design is successful the film’s universe feels natural, immersing the audience in the story. So use the principles wisely and may the forces be with you.

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Biosketch

Professor Alejandro Garcia has taught *Physics of Animation* at San Jose State since 2009. In 2011 he spent a one-year professional leave as physicist-in-residence at Dreamworks Animation SKG. During his time with Dreamworks he worked in the Division of Artistic Development, presenting over 30 classes and special lectures for various studio departments and consulted on *Madagascar 3: Europe’s Most Wanted*. This work was highlighted on Science Nation; see: <http://tinyurl.com/8ynw2tt>. Dr. Garcia is also the author of the textbook, *Numerical Methods for Physics*, and has published over 80 professional articles in the fields of computational physics, statistical mechanics, and fluid mechanics.

Image credits

Illustrations by Katie Corna: Figures 16(left), 22(right); Charlene Fleming: Figures 1, 2(left), 3(right), 4(right), 6(left), 8, 9(left), 14(right), 15, 17(left,right), 18; Stephanie Lew: Figures 21(right), 23; Dora Roychoudhury: Figures 6(right), 13, 25; Rebbaz Royee: Figures 3(left), 4(left), 5, 7, 14(left,center), 17(center); Corey Tom: Figures 9(right), 10, 12, 16(center), 19, 20, 21(left), 22(left). All photos are by the author.